Dynamic Compensation under Uncertainty Shocks and Limited Commitment: Internet Appendix

- For On-line Publication Only

January 2017

This Online Appendix begins with a formal characterization of the main implications of the paper. I then discuss two extensions. First, I address the implementation of the limited commitment contract, which justifies the limited commitment constraint by revealing a similarity between a firm's commitment to a contract termination time and its commitment to a capital structure. Two, I explore the equilibrium in which shirking is optimal and discuss its implications for both empirical studies and policy recommendations.

I Formal Analysis of Volatility Regime-switching under Limited Commitment

Here, I characterize the dynamics of compensation following uncertainty shocks. Given any W_{t^+}, the agent's continuation utility after the volatility increase, the goal is to characterize the distribution of the agent's wealth after a certain amount of time elapses. Given the property of Brownian motions, it is equivalent to characterizing the amount of payment by the frequency of payment, which is the approach adopted in this section.

Following Cox and Miller (1977), given the dynamics of W, the transition density function $f(t, W; W_{t^+})$ for a process starts with W_{t^+} and satisfies the Kolmogorov forward equation

$$\frac{\partial}{\partial t} f(t, W; W_{t^+}) = \frac{1}{2} \frac{\partial^2}{\partial W^2} \left[\lambda^2 \sigma_h^2 f(t, W; W_{t^+})\right] - \frac{\partial}{\partial W} \left[\gamma W f(t, W; W_{t^+})\right] \ ,$$

subject to boundary conditions

$$f(t, R; W_{t^+}) = 0 \text{ and}$$

$$\frac{1}{2} \frac{\partial}{\partial W} \left[\lambda^2 \sigma_h^2 f(t, W; W_{t^+})\right] |_{W=\overline{W}_h} - \gamma \overline{W}_h f(t, \overline{W}_h; W_{t^+}) = 0 \ .$$

Unfortunately, this partial differential equation is generally intractable. However, when γ is small, the dynamics of W can be approximated by a standard Brownian motion with one absorbing boundary R and one reflecting boundary $\overline{W}_h$, the transition density of which has an explicit form.[1] The approximation follows the method developed in Ward and Glynn (2003). After obtaining the transition density, I can measure the likelihood of cash payments given a certain time period T after the shock using the concept of local time in stochastic processes. Given a time period T and initial point W_{t^+}, define local time $\mathcal{J}$ as follows:

$$\mathcal{J}_h(T; W_{t^+}) = \lim_{\varepsilon \to 0} \frac{1}{2\varepsilon} \int_0^T \mathbb{1}_{\left\{\overline{W}_h - \varepsilon < W_t < \overline{W}_h + \varepsilon\right\}} dt | W_0 = W_{t^+} \ ,$$

where $\mathbb{1}_{\{.\}}$ is the indicator function. This local time is a random variable that measures the amount of time W spends in the neighborhood of the payment boundary. Since being at the payment boundary implies cash payments, this can be interpreted as the frequency of payments an agent with initial wealth W_{t^+} receives within time T after the economy enters crisis mode.

[1]The assumption $\gamma > r$ is still needed for the benchmark full commitment contract to exist. It is not necessary, though, for the limited commitment contract. See the end of Section 2 in the main paper for the related discussion.

The definition of $\mathcal{J}_h$ allows the following proposition:

Proposition I.1. *Assume γ is small. There exists $\widehat{T}$ and $\widehat{W}_{t^-}$ such that if $T < \widehat{T}$*

$$E^L\left[\mathcal{J}_h(T; W_{t^-})\right] > E\left[\mathcal{J}_h(T; W_{t^-})\right] \ \ \textit{and}$$

$$E^L\left[\mathcal{J}_h(T; W_{t^-})\right] > E^L\left[\mathcal{J}_l(T; W_{t^-})\right]$$

for all $W_{t^-} > \widehat{W}_{t^-}$, where E^L represents the expectation under the limited commitment contract.

Proposition I.1 is a formal characterization of the observations obtained in Section 3 of the main paper. Despite the mathematical complexity, its basic intuition is quite simple: first, comparing the limited and full commitment contract, W is closer to the payment boundary after the uncertainty shock under the limited commitment contract. When γ is small, the process of W behaves similarly to a standard Brownian motion and thus spends more time at the payment boundary whenever the starting point is closer to the boundary. In more intuitive terms, the agent should expect more frequent payments in the near future if his cumulative performance is closer to the target bonus hurdle set by his contract. The similar argument applies to the comparison between the limited commitment contract in low and high volatility states: W_{t^+} is closer to $\overline{W}_h^L$ than W_{t^-} is to $\overline{W}_l^L$.

Why does Proposition I.1 hold only when T is small? This is because while W_{t^+} is closer to the payment boundary after the shock under the full commitment contract, it is also closer to the termination boundary because the agent is overall punished. As T increases, the likelihood of contract termination rises faster for the limited commitment contract. That is, agents now operate under tighter financial slack. The longer into a crisis, the more likely is termination, as the possibility of realizing a series of losses becomes more real. The conclusion in Proposition I.1 thus holds only for T small enough, when the probability of termination is negligible. As shown by the numerical simulations, this pertains to the second observation that cash payment vanishes very quickly under the high volatility state under limited commitment. The notion of termination likelihood can be formally described using the concept of stopping time, as the next proposition shows.

Proposition I.2. *Define* $\tau_s = \inf\left\{t : W_t = R|\overline{W}_s\right\}$ *as the termination time given payment threshold* $\overline{W}_s$. *Then,*

$$E^L(\tau_h) < E^L(\tau_l) \text{ and}$$

$$E^L(\tau_h) < E(\tau_h).$$

When the commitment constraint is binding, the agent's expected termination time is shorter under high volatility.

Intuitively, given the absorbing boundary R and reflecting boundary $\overline{W}_h$, a process with initial value W_{t^+} is in expectation stopped earlier whenever W_{t^+} is closer to R and $\overline{W}_h$ is smaller. The limited commitment contract satisfies both conditions. Further, it should be noted that this proposition does not require the assumption of a small γ, as the expected speed of growth for W is lower when W_{t^+} is lower, which the limited commitment contract again satisfies. Nevertheless the proof of Proposition I.2 still imposes the restriction on γ for the sole purpose of analytical tractability.

The results of this subsection imply that the recipients of crisis time bonuses are those who perform relatively well before the crisis. Proposition I.1 states that more frequent cash compensation is conditional on the agent's wealth before the shock W_{t^-} surpassing a certain threshold, and higher W_{t^-} represents better before-shock performance. Those who perform relatively poorly ex ante are no longer around after the crises as a result of either replacement or firm liquidation. This suggests that those who produce the largest profits before the crisis are being criticized the most for receiving bonuses during the crisis. One should keep in mind, however, that the huge loss of firm wealth is primarily due to the risky aggregate environment and, despite receiving bonuses for a short period into the crisis, managers are hurt overall.

II Contract Implementation, Capital Structure and the Commitment Constraint

The results from the main paper highlight the different dynamics of compensation generated by full versus limited commitment contracts. In this section, I explore their different implementations and establish a novel equivalence between firms' commitment to compensation contracts and their commitment to capital structure. This equivalence provides a justification for the limited commitment assumption made throughout this paper, as firms' commitment to capital structure is known to be implausible.

Implementing the full commitment contract involves the use of debt and equity and therefore creates a conflict between debt and equity holders that leads to potential commitment issues. When the regime switches from low to high volatility, the face value of debt must be brought down at the expense of equity holders. Such implementation imposes an implicit assumption that equity holders must commit to maintaining a certain capital structure, which is generally implausible since equity holders do not always act for the benefit of the entire firm. In contrast, contracts with limited commitment do not require firms to make such commitment to capital structure and should therefore be more prevalent in practice.

Implementation in this paper follows the standard literature in using a set of common securities with limited liability: equity, long-term debt and credit line. Equity can be held by both the manager as well as outside investors who receive dividend payments and can decide the firm's capital structure. The long-term debt is a callable consol bond that pays a fixed rate and has a fixed face value. The firm can issue more long-term debt or call it back at its face value.[2] Finally, the credit line provides the manager with limited liquidity. The

[2] Although callable debt is usually redeemed at a premium, the specific value of the premium does not play a role in this model and is therefore without loss of generality assumed to be zero.

manager decides both the dividend and the credit line balance, but incentive compatibility under the optimal contract renders irrelevant who makes dividend and credit line decisions.

There is more than one implementation of the optimal contract. The following proposition provides a standard result:

Proposition II.1. *Both the full and limited commitment contract can be implemented by*

(a) the manager holding inside equity share λ*;*

(b) the face value of the callable debt satisfying $D_s = V_s(\overline{W}_s)$*;*

(c) the credit line balance M_t *and credit limit* C^* *satisfying* $W_t = \lambda(C_s^* - M_t)$ *and* $\lambda C_s^* = \overline{W}_s$*.*

Dividend is paid when $M_t = 0$*. Liquidation occurs when* M_t *reaches* C_s^**.*

The implementation is intuitive and hence the explanation here concise. Since λ measures the portion of private benefit the manager can derive from shirking, its value represents the least degree of sensitivity to cash flow to which the manager is exposed. Managers can draw down the line of credit for operating liquidity. Dividends serve as reward to the manager as well as returns to outside investors. Since the manager has a higher discount rate, dividends will not be saved inside the firm as long as the credit line is paid in full. Finally, the amount of long-term debt is used to adjust the profit rate of the firm such that incentives remain unchanged.

Security implementation of the optimal contract implies a certain capital structure which can potentially raise questions under the regime switching environment: the boundary conditions of the full commitment contract implies $V_h(\overline{W}_h) < V_l(\overline{W}_l)$. Since $V_h(\overline{W}_h)$ and $V_l(\overline{W}_l)$ also correspond to the face value of the callable debt in the high and low volatility states, the implementation of the full commitment contract requires that the face value of long-term debt be brought down when volatility increases. In other words, some portion of the long-term debt must be called back, hence the usage of callable debt here. These callbacks induce a transfer of wealth out of equity holders' pockets while debt holders are paid in full.

To further investigate this problem, I compute the value of the aforementioned securities, in particular that of equity. To simplify the analysis, I assume that $L = 0$, so that there will be no residual value in the event of firm liquidation, eliminating the need to specify the priority of residual claims among equity holders and debt holders. Let V_s^E denote the equity value, which is defined by

$$V^E(W_t) = E\left(\int_t^\tau e^{-rs} dDiv_s | W_t\right) ,$$

where W_t, the manager's continuation utility, can be transferred to M_t, the credit line balance, through the relationship defined in Proposition 6.

The value function of equity value can be characterized by the following differential equation:

$$rV_s^E(W) = (\gamma W - \pi_s \delta_s(W)) V_s^{E\prime} + \frac{1}{2}\lambda^2 \sigma_s^2 V_s^{E\prime\prime} + \pi_s (V_{\widehat{s}}^E (W + \delta_s(W)) - V_s^E(W))$$

subject to boundary conditions

$$V_s^E(0) = 0 ;$$

$$V_s^{E\prime}(\overline{W}_s) = 1 ;$$

where $V_{\widehat{s}}^E$ is the value of equity in state $\widehat{s}$. The implementation requires equity holders to commit to the particular capital structure specified in the optimal contract by redeeming the outstanding debt at the time of the uncertainty shock.

Do equity holders always find it preferable to recall debt when uncertainty is high? The answer is hardly yes, as equity holders can usually withdraw investment in practice and default on any debt obligation. In this model, let $D_{\widehat{s}} - D_s$ measure the value of debt redemption. Equity holders will find it optimal to default when $V_{\widehat{s}}^E$, the value from maintaining the firm, is lower than $(D_{\widehat{s}} - D_s)$, the cost of doing so. On the contrary, under the limited commitment contract, $V_s(\overline{W}) = L$ for both $s = l$ and h implies an identical face value of long-term debt before and after regime switching. That is, the capital structure of the limited commitment contract can be maintained without a tendency on the part of equity holders to default ex post.

Equity holders making ex post default decisions is common in both financial research and in practice. There is a large body of literature studying the endogenous default decision of equity holders and the conflict with debt holders, notably Leland (1994), Leland and Toft (1996) and He and Xiong (2012). In this paper, I do not characterize the exact default boundary of equity holders; rather, the point I want to make is that equity holders cannot (credibly) commit to not defaulting for the sake of the entire firm when there is a chance of their finding default preferable ex post.

In additional to justifying the prevalence of limited commitment contracts in practice, the equivalence between the commitment to contract termination time and the commitment to capital structure also offers an empirically testable hypothesis: the investors of more distressed firms are more likely to withdraw their investment, default on firms' debt and alter firms' capital structure. The degree of distress can be a potential proxy for the commitment power firms have over their labor contracts which is difficult to observe.

III Optimal Compensation with Shirking

Throughout the previous analyses, it has been assumed that working is always preferred by the principal regardless of the level of uncertainty. This section relaxes this assumption and examines when the optimal contract allows shirking in equilibrium. The results carry both policy and empirical implications. When the contract allows shirking during crisis times, no bonuses are paid. This may appear agreeable to policymakers and to public sentiment, but it is actually worse because the average productivity of the economy is lower as a result of lower managerial effort. Empirically, studies of compensation and performance, such as those examining pay-performance sensitivity, could be confounded by the endogeneity between return and volatility driven by unobservable changes in managerial effort.

Which effort level is optimal in equilibrium depends on the cost of allowing shirking. Working is preferred as long as C, the social cost of shirking measured by the reduction in average cash flow, is high. This section explores this assumption in more detail. Consider a contract that involves no payment but simply allows the agent to shirk forever. Define (W^S, V^S) as the pair of payoffs for the agent and the principal respectively if the agent exerts effort $e_t = \underline{e}$ for all t. Then,

$$W^S = \frac{\lambda C}{\gamma} \; ;$$

$$V^S = \frac{1}{r}\left(\mu - \frac{\lambda C}{\gamma}\right) \; . \tag{1}$$

Notice that this payoff is a function of A, which is an irrelevant variable in the incentive compatible contract characterized in Propositions 1 and 2. Therefore, whether the incentive compatible contract is optimal for the principal depends on the level of V^S. When C is sufficiently low, $V^S > V(W^*)$, where $V(W^*) \equiv \max V(W)$ is the maximal value the principal can derive from an incentive compatible contract, the principal is better off stopping incentive provision.[3] The agent will choose to shirk, receive no payment from the principal and instead be compensated by his private benefit from shirking. The optimal contract is static, unrelated to the agent's performance and therefore involves no termination.

The different maximal firm value under low and high volatility states raises the possibility that working is not always optimal for both states. If $V_l^* > V^S > V_h^*$, the optimal contract will induce working as long as $s = l$ and switches to the static contract at times when $s = h$. In the model where only one state transition occurs, the dynamics of the optimal contract follow the ODEs described in Proposition 2, except the value function V_h is replaced by the

[3]Strictly speaking, the contract that allows shirking forever is optimal only when $V^S > B(W)$, where $B(W)$ is a V-shaped function that extends above V_h^*. See Zhu (2013) for detail. Here, I avoid the complicated situations where V^S lies above b^* but below $B(W)$ by assuming that C is either high enough or low enough such that either working or shirking permanently is the optimal effort.

static payoff given by equation (1).

If shirking is optimal in the high volatility state, the procedure that pins down $\delta_l(W)$ is slightly different. Under the static contract that allows shirking when $s = h$, the agent's continuation utility is a singleton W^S. Jumps of W from the low to the high volatility state is simply $\delta_l(W) = W^S - W$. Note that W^S measures not the discounted future income but the present value of private shirking benefit to the agent. This value is automatically achieved as long as the principal immediately ceases any payment. Moreover, because W^S is no longer sensitive to the agent's performance, there is no contract termination after the regime switches to high volatility. All firms survive regardless of their agent's performance history up to the regime switching time. The next proposition summarizes these findings:

Proposition III.1. *Suppose C is low or σ_h is sufficiently high, the optimal contract induces $e_t = \bar{e}$ under σ_l but $e_t = \underline{e}$ under σ_h. The principal's value function $V_l(W)$ and payment boundary $\overline{W}_l$ satisfy*

$$rV_l(W) = \mu + (\gamma W - \pi_l \delta_l(W))\, V_l'(W) + \frac{1}{2}\lambda^2 \sigma_L^2 V_l''(W) + \pi_l \left(V_h - V_l(W)\right) \ ,$$

subject to boundary conditions $V_l(R) = L$, $V_l'(\overline{W}_l) = -1$ and $V_l''(\overline{W}_l) = 0$. Furthermore, $\delta_l(W) = W^S - W = \frac{\lambda C}{\gamma} - W$, and V_h is given by

$$V_h = V^S = \frac{1}{r}\left(\mu - \frac{\lambda C}{\gamma}\right) \ .$$

The existence of an optimal contract that involves shirking in the equilibrium has important policy implications. Since the manager is compensated through the private benefit of shirking when uncertainty is high, no cash payment is made under that regime. This implies the possibility of observing little or no bonuses during a recession. However, although to the media's or the public's liking, this equilibrium is actually worse in terms of total welfare, because productivity, measured by mean cash flow, is now lower due to less effort from managers. This is true as long as $\lambda < 1$, i.e. when there is deadweight loss associated with managerial shirking. This result highlights the importance of compensation in keeping mangers properly incentivized, even though the exact timing of their compensation may not match their overall performance at the time when a large negative shock occurs.

The shirking equilibrium also reveals a potential endogeneity problem between profitability and volatility. Existing empirical work that studies compensation often considers profitability and volatility as independent factors. However, fluctuation in profitability can be driven by changes in volatility through the channel of managerial effort, raising empirical challenges since such effort is normally difficult to measure. It also provides further evidence in addition to previous work that uncertainty is the key to understanding the recent financial crisis.

It is worth noting that the change in the agent's equilibrium effort is a feature of increasing volatility but not necessarily of decreasing profitability, which sets this paper apart from those with similar regime switching techniques such as Hoffmann and Pfeil (2010). While lower average cash flow μ does bring down firm value under an incentive compatible contract, it also lowers V^S, firm value under a static contract that allows shirking. As a result, working can still be the optimal effort to induce if $V^S < V_h^*$. In contrast, V^S does not depend on σ, but V_h^* does. When cash flow volatility becomes higher, V_h^* becomes lower until falling below V^S, and the incentive compatible contract is dominated by the static contract, a unique feature of stochastic volatility.

IV Proofs

Proof of Proposition I.1:

Following Cox and Miller (1977), the transition density of the process W in the high variance state given initial value W_{t^+} follows the Kolmogorov forward equation:

$$\frac{\partial}{\partial t}f(t,W;W_{t^+}) = \frac{1}{2}\frac{\partial^2}{\partial W^2}\left[\lambda^2\sigma_h^2 f(t,W;W_{t^+})\right] - \frac{\partial}{\partial W}\left[\gamma W f(t,W;W_{t^+})\right] \ ,$$

subject to the boundary conditions

$$f(t,0;W_{t^+}) = 0 \ ;$$

$$\frac{1}{2}\frac{\partial}{\partial W}\left[\lambda^2\sigma_h^2 f(t,W;W_{t^+})\right]|_{W=\overline{W}_h} - \gamma\overline{W}_h f(t,\overline{W}_h;W_{t^+}) = 0 \ ,$$

where f is a density function conditional on $W_{t^+} = W$.

Define $\sigma^2 = \lambda^2\sigma_h^2$ as the overall variance of the W process. Let f_γ be the solution to this boundary value problem for a particular γ. According to Ward and Glynn (2003), when γ is closer to zero, f_γ can be approximated by

$$f_\gamma(t,W;W_{t^+}) = k(\gamma)g(t,W;W_{t^+}) + o(\gamma) \ , \tag{2}$$

where $k(\gamma) = \left(1 - \frac{\gamma}{2\sigma^2}W_{t^+}^2 + \frac{\gamma}{2\sigma^2}W^2 + \frac{\gamma}{2}t\right)$ and g is the corresponding transition density function for the same process but with $\gamma = 0$.

Now the problem becomes a Brownian motion between an absorbing and a reflecting barrier. In particular, $g(t,W;W_{t^+})$ satisfies the differential equation:

$$\frac{\partial}{\partial t}g(t,W;W_{t^+}) = \frac{1}{2}\frac{\partial^2}{\partial W^2}\left[\sigma^2 g(t,W;W_{t^+})\right] \ ,$$

subject to boundary conditions $g(t,R;W_{t^+}) = 0$ and $\frac{1}{2}\sigma^2\frac{\partial}{\partial W}[g(t,W;W_{t^+})]|_{W=\overline{W}_h} = 0$.

The solution to this problem has been derived by Schwarz (1992) as

$$g(W,t) = \sum_{n=1}^{\infty} A_n \exp\left(-\alpha_n^2\frac{1}{2}\sigma^2 t\right)\cos\left(\alpha_n W\right) \ ,$$

where $\alpha_n = \frac{(2n-1)\pi}{2\overline{W}_h}$ and $A_n = \frac{\cos\left(\alpha_n W_{t^+}\right)}{\overline{W}_h}$.

Substituting this into the approximation function (2) yields $f(W,t)$, which can be used in the definition of the expected local time at the payment boundary:

$$E\left[L_h(T;W_{t^+})\right] = \lim_{\varepsilon\to 0}\frac{1}{2\varepsilon}\int_0^T dt\int_{\overline{W}_h-\varepsilon}^{\overline{W}_h+\varepsilon} f(t,W;W_{t^+})dW$$

Fix some $W_{t^+} < \overline{W}_h^L$. Let

$$E[\mathcal{J}_h(T;W_{t^+})] \equiv E[\mathcal{J}_h(T;W_{t^+})|\overline{W}_h]$$

be the expected local time given the full commitment value functions and payment boundaries and

$$E^L[\mathcal{J}_h(T;W_{t^+})] \equiv E[\mathcal{J}_h(T;W_{t^+})|\overline{W}_h^L]$$

be the expectation of local time at the payment boundary under the limited commitment contract. First, $\frac{\partial}{\partial T}E[\mathcal{J}_h(T;W_{t^+})]|_{T=0} > 0$; that is, the expected time spent at one point is longer whenever the time interval is longer, in particular when the time interval increases by a small amount from 0. Secondly, such derivative is larger for smaller $\overline{W}_h$ because for a fixed W, $f(W,t)$ is decreasing in $\overline{W}_h$. The effect of increasing the time interval is bigger, the shorter is the distance between W_{t^+} and the reflecting boundary. Note that in the case of $\sigma \gg \gamma$, the approximation adjustment term $h(\gamma)$ is close to one if W and W_{t^+} are near each other, this implies that the most precise approximation is around the payment boundary, exactly the target of the analysis given here.

Recall that $\overline{W}_h > \overline{W}_h^L$ and $E^L[\mathcal{J}_h(0;W_{t^+})] = E[\mathcal{J}_h(0;W_{t^+})] = 0$ implies

$$E^L\left[\mathcal{J}_h(T;W_{t^+})\right] > E\left[\mathcal{J}_h(T;W_{t^+})\right] \;\; as\ T \to 0\ .$$

The expected local time grows faster for closer reflecting boundary near $T = 0$. Also,

$$E^L\left[\mathcal{J}_h(T; W_{t^+})\right] < E\left[\mathcal{J}_h(T; W_{t^+})\right] \;\; \textit{as } T \to \infty \;,$$

which implies that there is some $\widehat{T}$ such that

$$E^L\left[\mathcal{J}_h(\widehat{T}; W_{t^+})\right] = E\left[\mathcal{J}_h(\widehat{T}; W_{t^+})\right] \;,$$

and

$$E^L\left[\mathcal{J}_h(T; W_{t^+})\right] > E\left[\mathcal{J}_h(T; W_{t^+})\right] \;\; \textit{for all } 0 < T < \widehat{T} \;.$$

Finally, notice that given $\overline{W}_h$, $E\left[L_h(T; W_{t^+})\right]$ is decreasing in W_{t^+}; that is, the further W_{t^+} is from the reflecting barrier, the less time it spends there within a certain time. Therefore, $E^L\left[\mathcal{J}_h(T; W^L_{t^+})\right] > E\left[\mathcal{J}_h(T; W_{t^+})\right]$ as long as $\overline{W}^L_h - W^L_{t^+} < \overline{W}_h - W_{t^+}$. Note that $\overline{W}^L_l - W^L_{t^+} < \overline{W}_h - W_{t^+}$ if $W_{t^-} > \widehat{W}$, and therefore $E^L\left[\mathcal{J}_h(T; W_{t^-})\right] > E\left[\mathcal{J}_h(T; W_{t^-})\right]$ for all $0 < T < \widehat{T}$ as long as $W_{t^-} > \widehat{W}$. $\square$

Proof of Proposition I.2:

Consider the process of W in the high volatility state with initial position W_{t^+}. Let N be the number of times W reaches the reflecting boundary $\overline{W}_h$ before it is stopped. Then

$$E\left[\tau\right] = \sum_{i=0}^{\infty} E\left[\tau, N = i\right] \;.$$

First, consider $N \geq 1$. If W reaches $\overline{W}_h$ at least once before it is stopped, then starting from $\overline{W}_h$, the expected stopping time is smaller whenever $\overline{W}_h - R$ is a shorter interval. Next, consider the case of $M = 0$. The expected stopping time is smaller whenever W_{t^+} is closer to R. Finally, the average speed of growth for W, γW, is slower for smaller W. Moreover, $E^L\left[\tau_h\right] < E\left[\tau_h\right]$, because $\overline{W}^L_h < \overline{W}_h$ and $W^L_{t^+} < W_{t^+}$ for the same W_{t^-}.

The same comparison can be made between $E^L[\tau_h]$ and $E^L[\tau_l]$. The expected stopping time is smaller when $\overline{W}$ and the initial W is closer to R, and when σ is larger.

The exact value of $E[\tau]$ is difficult to compute due to the irregular process W follows. However, when γ is small, the same approximation method used in the proof of Proposition I.1 can be applied here as well. The problem becomes a standard absorbing time question for a Brownian motion between an absorbing and a reflecting barrier, the solution to which is given by Cox and Miller (1977) as

$$E[\tau] = \frac{W_{t^+}(2\overline{W}_h - W_{t^+})}{\sigma^2} \ .$$

This solution confirms that $E[\tau]$ is positively related to $\overline{W}_h$ and W_{t^+} and negatively related to σ. Since $\overline{W}_h^L < \overline{W}_l^L < \overline{W}_h$, $W_{t^+}^L < W_{t^-}^L < W_{t^+}$ and $\sigma_h > \sigma_l$, $E^L[\tau_h]$ must be the smallest compare to $E[\tau_h]$ and $E^L[\tau_l]$. $\square$

Proof of Proposition II.1:

Proof: Without loss of generality, assume that the interest rate of the credit line is γ. Begin with the high volatility state σ_h. The credit line balance evolves according to

$$dM_t = \gamma M_t dt + xdt + dDiv_t - dY_t \ , \tag{3}$$

where Div_t represents the cumulative dividends paid by the firm and x is the consol bond rate. Substituting $x = rD_t$ into equation (3),

$$dWt = -\lambda dM_t = -\lambda\gamma Mtdt - \lambda xdt - \lambda dDiv_t + \lambda dY_t$$

$$= \gamma W_t dt - \lambda dI_t + \lambda(dY_t - \mu dt) \ ,$$

which satisfies incentive compatibility. The argument for state σ_L can be made analogously subject to a jump δ_L, the value of which is pinned down by the matching first order derivatives procedure. □

Proof of Proposition III.1:

Proof: This Proposition is a natural extension of the proofs of Propositions 1 and 2 in the main paper. The exact conditions under which shirking forever is optimal can be found in Zhu (2013). □

References

Cox, D. and Miller, H. 1977. *The theory of stochastic processes*. Chapman and Hall.

He, Z. and Xiong, W. 2012. Rollover risk and credit risk. *Journal of Finance*, 67:391–430.

Hoffmann, F. and Pfeil, S. 2010. Reward for luck in a dynamic agency model. *Review of Financial Studies*, 23:3329–3345.

Leland, H. E. 1994. Corporate debt value, bond covenants, and optimal capital structure. *Journal of Finance*, 49:1213–1252.

Leland, H. E. and Toft, K. B. 1996. Optimal capital structure, endogenous bankruptcy, and the term structure of credit spreads. *Journal of Finance*, 51:987–1019.

Schwarz, W. 1992. The wiener process between a reflecting and an absorbing barrier. *Journal of Applied Probability*, 65:597–604.

Ward, A. and Glynn, P. 2003. Properties of the reflected ornstein–uhlenbeck process. *Queueing Systems*, 44:109–123.

Zhu, J. Y. 2013. Optimal contracts with shirking. *Review of Economic Studies*, 80:812–839.